Tentamen Quantum Fysica I

March 9, 1999

Please **print** *your name, student number and complete address on the first page. Each problem is to be answered on a* separate *page. Print your name on top of each page.*

Elke opgave op en apart vel. Zet op het eerste vel duidelijk uw naam, student nummer en adres. Op elk volgend vel uw naam vermelden.

Problem 1.

(a) For an operator A, under what circumstance is it Hermitian (i.e. $A = A^{+}$) ?

(b) Write the Time Dependent Schrödinger Equation. What is meant by a "stationary state"?

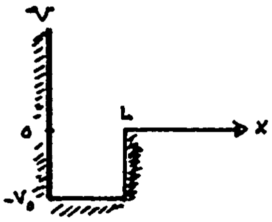
(c) Why are "quadratically integrable" (or "square integrable") functions important in quantum mechanics?

QF-II → in 1999/2000 (d) What is meant by the "Schrödinger picture" and by the "Heisenberg picture" ?

Problem 2.

A particle of mass m is confined to move in one dimension by a potential $V(x)$

$$V(x) = \begin{cases} +\infty & x < 0 \\ -V_0 & 0 < x < L \\ 0 & L < x \end{cases}$$



(a) Derive an equation for the bound state.

(b) From the result of part (a), derive an expression for the minimum value of V_0 which will have a bound state.

Problem 3.

A particle is confined to move in a *symmetric* potential $V(x) = V(-x)$ all of whose states are bound. At $t = 0$ the state of the system is

$$\Psi(x,0) = N[u_0(x) + u_1(x) + u_2(x) + u_3(x) + u_4(x)].$$

u_0 is the normalized ground state wave function, and u_n are the normalized excited state $(n = 1, \ldots, 4)$ wave functions corresponding to the energies $E_n, n = 0, \ldots, 4$. The parity of $u_n(x)$ is $(-1)^n$.

(a) Choose N so that Ψ is normalized.

(b) Write the normalized wave function for $t > 0$.

(c) What is the expectation value of the parity for a system in the state $\Psi(x,0)$?

(d) At some time the parity is measured. What is the probability that the result is $+1$?

(e) Suppose that the result of a parity measurement is $+1$. Write the normalized wave function for time t subsequent to this measurement.

(f) Subsequently the energy is measured. What is the probability that the outcome of this measurement is E_0?

(g) What is the expectation value of the the energy for the system described by $\Psi(x,0)$ if the Hamiltonian is that of the harmonic oscillator?

(h) Consider, again, that the Hamiltonian is that of the harmonic oscillator, and that a parity measurement resulted in the value $+1$. What is the expectation value of the energy after this measurement?

(i) In this last part we make *no* assumptions about the symmetry of $V(x)$.

Suppose that at $t = 0$ the system is in a normalized state $\Psi(x,0)$. The parity is measured at $t = 0$ and the result is -1. Write the (new) normalized wave function immediately after the parity measurement.

Problem 4.

Two operators, A and B, satisfy the equations

$$A = B^+B + 3$$
$$A = BB^+ + 1.$$

(a) Show that A is self-adjoint, $A^+ = A$.

(b) Find the commutator $[B^+, B]$.

(c) Find the commutator $[A, B]$.

(d) Suppose ψ is an eigenfunction of A with eigenvalue a:

$$A\psi = a\psi$$

Show that if $B\psi \neq 0$ then $B\psi$ is an eigenfunction of A, and find the eigenvalue.

Problem 5.

In this problem we show that, for an arbitrary *free particle* wave packet,

$$< x >_t = < x >_0 + \frac{< p >_0}{m} t \tag{1}$$

(a) Equation (1) is an example of a "principle". Which principle? Give a brief statement of the principle.

You are asked to obtain (1) in two ways:

(b) Use the Heisenberg equation

$$i\hbar \frac{dx}{dt} = [x, H].$$

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(c) Work in momentum space. Express $\phi(p,t)$ in terms of $\phi(p,0)$ and express the operator x in terms of $\frac{d}{dp}$ and evaluate expectation values in momentum space.